数恋教情

# 发现规律　继续追击

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## §1. 问题源起

**导言**

单墫教授在短文《发现规律》[1] 中发布一个问题 (下称问题 1):

**问题 1.**

计算

$$
\begin{aligned}
&1^3+5^3+3^3=?\\
&16^3+50^3+33^3=?\\
&166^3+500^3+333^3=?\\
&1666^3+5000^3+3333^3=?\\
&\cdots\cdots
\end{aligned}
$$

你看到什么? 你能说出你的一般结果, 并加以证明吗?

**概要**

费振鹏先生写了《如何发现规律》[2] 一文, 给出下述公式并给予证明:

$$1\underbrace{6\cdots6}_{n-1\text{个}6}{}^3+5\underbrace{0\cdots0}_{n-1\text{个}0}{}^3+\underbrace{3\cdots3}_{n\text{个}3}{}^3=1\underbrace{6\cdots6}_{n-1\text{个}6}5\underbrace{0\cdots0}_{n-1\text{个}0}\underbrace{3\cdots3}_{n\text{个}3}. \tag{1}$$

## §2. 尾数微调

**概要**

笔者把 $16\cdots6$ 和 $3\cdots3$“尾数微调”, 改为 $\overline{16\cdots6a}$ 和 $\overline{3\cdots3b}$, 类似地可得到一系列公式, 证明参照 [2] 不难, 详略:

$$1\underbrace{6\cdots6}_{n\text{个}6}{}^3+5\underbrace{0\cdots0}_{n\text{个}0}{}^3+\underbrace{3\cdots3}_{n\text{个}3}2^3=1\underbrace{6\cdots6}_{n\text{个}6}1\underbrace{6\cdots6}_{n-1\text{个}6}8\underbrace{6\cdots6}_{n\text{个}6}4; \tag{2}$$

$$1\underbrace{6\cdots6}_{n\text{个}6}{}^3+5\underbrace{0\cdots0}_{n\text{个}0}{}^3+\underbrace{3\cdots3}_{n\text{个}3}1^3=1\underbrace{6\cdots6}_{n-1\text{个}6}58\underbrace{3\cdots3}_{n-1\text{个}3}8\underbrace{9\cdots9}_{n-1\text{个}9}87; \tag{3}$$

$$1\underbrace{6\cdots6}_{n\text{个}6}{}^3+5\underbrace{0\cdots0}_{n\text{个}0}{}^3+\underbrace{3\cdots3}_{n\text{个}3}4^3=1\underbrace{6\cdots6}_{n\text{个}6}8\underbrace{3\cdots3}_{n-1\text{个}3}4\underbrace{0\cdots0}_{n+1\text{个}0}; \tag{4}$$

$$1\underbrace{6\cdots6}_{n\text{个}6}{}^3+5\underbrace{0\cdots0}_{n\text{个}0}{}^3+\underbrace{3\cdots3}_{n\text{个}3}5^3=1\underbrace{6\cdots6}_{n-1\text{个}6}71\underbrace{6\cdots6}_{n-1\text{个}6}9\underbrace{6\cdots6}_{n-1\text{个}6}71; \tag{5}$$

$$1\underbrace{6\cdots6}_{n\text{个}6}7^3+5\underbrace{0\cdots0}_{n+1\text{个}0}{}^3+\underbrace{3\cdots3}_{n+1\text{个}3}4^3=1\underbrace{6\cdots6}_{n+1\text{个}6}91\underbrace{6\cdots6}_{n-1\text{个}6}71\underbrace{6\cdots6}_{n\text{个}6}7; \tag{6}$$

$$1\underbrace{6\cdots6}_{n-1\text{个}6}8^3+5\underbrace{0\cdots0}_{n\text{个}0}{}^3+\underbrace{3\cdots3}_{n\text{个}3}6^3=1\underbrace{6\cdots6}_{n-1\text{个}6}7\underbrace{6\cdots6}_{n-1\text{个}6}74\underbrace{6\cdots6}_{n-1\text{个}6}88. \tag{7}$$

感兴趣的读者可自行尝试

$$\overline{1\underbrace{6\cdots6}_{n-1\text{个}6}a}^3+\overline{5\underbrace{0\cdots0}_{n\text{个}0}}^3+\overline{\underbrace{3\cdots3}_{n\text{个}3}b}^3\quad(n\geqslant1,\ a,b\in\{0,1,\cdots,9\}) \tag{8}$$

的情形, 得到相应的结果.

## § 3. 从立方到平方

**概要**

立方可以, 平方可以吗? 试试便知, 平方更可以:

$$1\underbrace{6\cdots6}_{n-1\text{个}6}{}^2+5\underbrace{0\cdots0}_{n-1\text{个}0}{}^2+\underbrace{3\cdots3}_{n\text{个}3}{}^2=3\underbrace{8\cdots8}_{n-1\text{个}8}\underbrace{4\cdots4}_{n-1\text{个}4}5; \tag{9}$$

$$1\underbrace{6\cdots6}_{n\text{个}6}{}^2+5\underbrace{0\cdots0}_{n\text{个}0}{}^2+\underbrace{3\cdots3}_{n\text{个}3}2^2=3\underbrace{8\cdots8}_{n-1\text{个}8}\underbrace{7\cdots7}_{n\text{个}7}80; \tag{10}$$

$$1\underbrace{6\cdots6}_{n-1\text{个}6}7^2+5\underbrace{0\cdots0}_{n\text{个}0}{}^2+\underbrace{3\cdots3}_{n\text{个}3}4^2=3\underbrace{8\cdots8}_{n-1\text{个}8}9\underbrace{4\cdots4}_{n\text{个}4}5; \tag{11}$$

$$1\underbrace{6\cdots6}_{n\text{个}6}8^2+5\underbrace{0\cdots0}_{n+1\text{个}0}{}^2+\underbrace{3\cdots3}_{n+1\text{个}3}6^2=3\underbrace{8\cdots8}_{n-1\text{个}8}9\underbrace{1\cdots1}_{n+1\text{个}1}20; \tag{12}$$

$$\cdots\cdots$$

感兴趣的读者不妨“展开火力”, 继续追击! 笔者早年的一篇习作 [3] 亦可供参考.

**参考文献**

[1] 单　墫. 发现规律 [J]. “单谈数学”公众号, 2023-03-15.

[2] 费振鹏. 如何发现规律 [J]. “数学风”公众号, 2023-03-16.

[3] 吴　康. 一类有规律的平方数 [J]. 中学数学研究 (广州), 1982 年第 5 期, P31 ～ 34.

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