怎样解题

吴伟朝先生十一月献题 (同余方程) 解答

问题

(1) 解同余式: $457x \equiv 458 \pmod{330625}$;

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解析

解: (1) 考虑

$$\begin{aligned} & 330625 \mid 457x - 458 \\ \Longleftrightarrow\ & 330625 \mid 457x - 458 + 330625y. \end{aligned}$$

而 $330625 = 457 \times 723 + 214$, 且 457 是质数, 为约去 457, 故只要

$$457 \mid 214y - 458 \iff 457 \mid 214y - 1.$$

令 $214y = 457z + 1$, 则

$$y = \frac{457z+1}{214} = 2z + \frac{29z+1}{214}.$$

令 $29z = 214t - 1$, 则

$$z = \frac{214t-1}{29} = 7t + \frac{11t-1}{29} = 7t + 3 + \frac{11(t-8)}{29}.$$

取 $t = 8$, 则 $z = 59$, $y = 126$. 所以

$$330625 \mid 457x - 458 + 330625 \times 126.$$

$$330625 \mid 457(x + 91156).$$

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$$x \equiv 239469 \pmod{330625}.$$

此即为所求同余方程的解.

(2) 这与 (1) 如出一辙. 设

$$190109375 \mid 457x - 458 + 190109375y.$$

因为 $190109375 = 457 \times 415994 + 117$, 所以只要

$$457 \mid 117y - 458 \iff 457 \mid 117y - 1.$$

令 $117y = 457z + 1$, 则

$$y = \frac{457z+1}{117} = 4z - \frac{11z-1}{117}.$$

令 $11z = 117t + 1$, 则

$$z = \frac{117t+1}{11} = 10t + \frac{7t+1}{11} = 10t + 2 + \frac{7\left(t-3\right)}{11}.$$

取 $t = 3$, 则 $z = 32$, $y = 125$. 所以

$$190109375 \mid 457x - 458 + 190109375 \times 125.$$

$$190109375 \mid 457\left(x + 51999281\right).$$

$$190109375 \mid x + 51999281.$$

$$x \equiv 138110094 \pmod{190109375}.$$

此即为所求同余方程的解. □